Trimmed Drag Considerations

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The distribution of lift between the wing and tail surfaces of a conventional aircraft is examined in order to determine that which would produce the minimum drag for a given lift. Further, the center of gravity position is determined which gives the desired lift distribution and, at the same time, maintains aircraft trim. Analytical expressions are developed which clearly show the dependence of this optimal c.g. position on the various aerodynamic and geometric parameters. In particular, it is shown how large changes in position of the optimal c.g. position can occur for small changes in the tail downwash angle. Numerical results are presented for a small twin-engine aircraft.

Nomenclature

AR_w, AR_t	= aspect ratio wing, tail
a_{wb}	= wing-body lift curve slope
$C_D, C_{D_{0wb}}, C_{D_{0t}}$	= drag coefficient, zero lift wing-body and tail
C_{D_0}	$= C_{D_{0wb}} + C_{D_{0t}} E_t$
$C_L, C_{L_{wb}}, C_{L_t}$	= lift coefficient, wing-body, tail
$(C_L)_t$	$= C_{L_t} E_t$
$C_m, C_{m_{0wb}}$	= pitching moment coefficient, zero lift wing-body
$\bar{c}$	= chord
E_t	= tail factor $(\eta_t S_t / S)$
h_0, h_{nwb}, h_t	= distance in chord lengths from leading edge of wing to c.g., wing-body aerodynamic center, tail
k_{wb}, k_t	= induced drag factor wing-body, tail
S, S_t	= wing and tail area
W	= weight
α, α_{0wb}	= angle of attack, zero lift wing-body
ϵ, ϵ_0	= downwash angle at tail, at zero lift wing-body
η_t	= tail efficiency factor

Introduction

THE continuing interest in fuel conservation as well as the desire to improve aircraft performance leads directly to the problem of reducing aircraft drag. To be more precise, it is of interest to reduce the trimmed drag or the drag of the vehicle in pitch equilibrium, since the aircraft is operated primarily in this mode. Typically, a reduction in wing-body or tail profile drag directly reduces the trimmed drag, and considerable efforts are made in this direction in aircraft design. Additional reductions, although small (1-5%), can be made by paying careful attention to the lift distribution between the wing-body and the tail surfaces (maintaining zero pitching moment) so as to obtain the proper tradeoffs in induced drag which minimize the overall drag. Typically, this aspect of the problem is called the trim drag problem.[1,2] Although it is this problem that will be treated here, only the overall trimmed drag is of interest.

One result which seems uniformly accepted is that the overall drag on an aft-tail vehicle can be reduced by moving the c.g. further aft, causing a reduced download on the tail or possibly even an upload.[3,4] Such a design is called a relaxed static stability design since the longitudinal stability is reduced considerably. This type of design is feasible if the desired level of stability can be maintained by a reliable control system. In fact, considerable interest has developed in designing aircraft with the knowledge that such control systems are present.[5,6] However, since such systems are expensive and have finite capacities, it seems appropriate to examine the possibility of reducing the trimmed drag in the basic design without stability augmentation or, if this is not possible, at least to identify the important parameters associated with the problem.

The purpose of this paper, therefore, is to examine the problem of reducing the trimmed drag of an aircraft by properly distributing the lift between the wing-body and the tail surfaces of the vehicle. It will be shown that there is a "best" distribution of lift that is dependent on certain wing-body and tail parameters. Furthermore, this distribution can be obtained by a proper c.g. location, which is determined by additional aerodynamic and geometric parameters. The c.g. location is sensitive to some of these parameters and can move forward, aft, or remain stationary as the trim lift coefficient changes. Calculations are performed for a small twin-engine aircraft.

Received Aug. 30, 1976.
Index categories: Configuration Design; General Aviation; Handling Qualities, Stability and Control.

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Analysis

Inherent in the subsequent development is the assumption that the aerodynamics are linear and can be represented as indicated. The required governing equations are (for small downwash)[2,5,7]

$$C_L = C_{L_{wb}} + C_{L_t} E_t = C_{L_{wb}} + (C_L)_t \tag{1}$$

$$C_m = C_{m_{0wb}} + C_{L_{wb}}(h_0 - h_{nwb}) - C_{L_t} E_t (h_t - h_0) = 0 \tag{2}$$

$$C_D = C_{D_{0wb}} + k_{wb} C_{L_{wb}}^2 + (C_{D_{0t}} + k_t C_{L_t}^2 + C_{L_t}\epsilon) E_t \tag{3}$$

Additional relations are given by

$$C_{L_{wb}} = a_{wb}(\alpha - \alpha_{0wb}) \tag{4}$$

$$\epsilon = (\partial\epsilon/\partial\alpha)(\alpha - \alpha_{0wb}) + \epsilon_0 \tag{5}$$

where ϵ_0 is the downwash angle at the tail at zero wing-body lift. Equation (1) can be used to eliminate C_{L_t} from Eq. (3), and Eqs. (4) and (5) used to eliminate ϵ from Eq. (3). The result is an expression for C_D in terms of $C_{L_{wb}}$ only. The value of $C_{L_{wb}}$ which minimizes C_D for a given C_L is determined by using simple calculus and is given by

$$C_{L_{wbopt}} = \frac{[2k_t' a_{wb} - \partial\epsilon/\partial\alpha] C_L + \epsilon_0 a_{wb}}{2[a_{wb}(k_{wb} + k_t') - \partial\epsilon/\partial\alpha]} \tag{6}$$

Where $k_t' = k_t / E_t$. The result given by Eq. (6) is valid regardless of whether the aircraft is trimmed or not, since Eq. (2) has not been used. Equations (1, 2, and 6) can be used to determine the c.g. position for minimum drag while retaining the aircraft in trim. The result is

$$h_{0_{\rm opt}} = \frac{[(2k_{wb}a_{wb} - \partial\epsilon/\partial\alpha)h_t + (2k_t'a_{wb} - \partial\epsilon/\partial\alpha)h_{nwb}]}{2[a_{wb}(k_{wb}+k_t') - \partial\epsilon/\partial\alpha]}$$

$$- \left\{ \frac{\epsilon_0 a_{wb}(h_t - h_{nwb})}{2[a_{wb}(k_{wb}+k_t') - \partial\epsilon/\partial\alpha]} + C_{m0wb} \right\} \frac{1}{C_L} \tag{7}$$

Finally, the minimum value of the drag coefficient for a given lift coefficient becomes

$$C_{D_{\rm opt}} = (C_{D_{0wb}} + C_{D_{0t}}E_t)$$

$$+ \left\{ \frac{4k_{wb}k_t'a_{wb} - (1/a_{wb})(\partial\epsilon/\partial\alpha)^2}{4[a_{wb}(k_{wb}+k_t') - \partial\epsilon/\partial\alpha]} \right\} C_L{}^2$$

$$+ \left\{ \frac{2k_{wb}a_{wb} - \partial\epsilon/\partial\alpha}{2[(k_{wb}+k_t')a_{wb} - \partial\epsilon/\partial\alpha]} \right\} \epsilon_0 C_L$$

$$- \frac{\epsilon_0{}^2}{4[k_{wb}+k_t' - (1/a_{wb})(\partial\epsilon/\partial\alpha)]} \tag{8}$$

Discussion

Equations (6-8) provide some insight into designing for minimum drag. Of particular importance is the effect of the parameter ϵ_0 on the results. The equations will be discussed in order.

The wing-body lift coefficient for minimum drag is given by Eq. (6). The form of the equation is

$$C_{L_{wb{\rm opt}}} = AC_L + B\epsilon_0 \tag{9}$$

with A and $B>0$. The optimal load distribution can be written in the form

$$(C_{L_{wb}}/C_L)_{\rm opt} = A + (B\epsilon_0/C_L) \tag{10}$$

or equivalently

$$((C_L)_t/C_L)_{\rm opt} = 1 - A - (B\epsilon_0/C_L) \tag{11}$$

Clearly if $\epsilon_0 = 0$, the optimal load distribution is constant for all values of C_L. In addition, it can be seen that an upload (download) on the tail is required for $A<1(A>1)$. For $\epsilon_0 \neq 0$, the optimum load distribution can require either up or download on the tail.

The possibilities are 1) $A>1$, $\epsilon_0>0$; 2) $A>1$, $\epsilon_0<0$; 3) $A<1$, $\epsilon_0>0$; and 4) $A<1$, $\epsilon_0<0$. Possibilities 2 and 3 allow a tail load reversal to occur whereas 1 and 4 require a down- and upload, respectively, for all C_L. The condition for zero tail load is given by

$$(2k_{wb}a_{wb} - \partial\epsilon/\partial\alpha) = \epsilon_0 a_{wb}/C_L \tag{12}$$

The c.g. position which insures trim as well as the optimal lift distribution between the wing-body and tail is given by Eq. (7), which can be written in the form

$$h_{\rm opt} = C - D/C_L \tag{13}$$

Equation (13) is analagous to the well-known relation for the trimmed aircraft

$$h_0 = h_n - C_{m0L}/C_L \tag{14}$$

where h_n and C_{m0L} are the aircraft neutral point and zero lift pitching moment, respectively. In general, one would expect C to be positive and D to be either positive or negative. For $D>0$, the c.g. position for minimum C_D will move *forward* as speed increases (C_L decreases) and vice-versa for $D<0$. The condition which must be satisfied for zero movement of the optimal c.g. throughout the flight range is given by the requirement

$$2[a_{wb}(k_{wb}+k_t') - \partial\epsilon/\partial\alpha]C_{m_{0wb}} + \epsilon_0 a_{wb}(h_t - h_{nwb}) = 0 \tag{15}$$

The importance of ϵ_0 in determining optimal c.g. position again is observed. If $\epsilon_0>0$ and sufficiently large, Eq. (15), the optimal c.g. position will move forward, Eq. (13), and there will be a download, Eq. (11), on the tail – an unusual result at first glance.

This result can be explained by examining Eq. (3). The term $C_{L_t}\epsilon$ is the culprit. Since the downwash tends to tilt the tail lift vector backward, a negative or downward lift would have a small component forward, reducing the overall drag.

The last equation to be discussed is that of the minimum drag coefficient, Eq. (8). The equation is quadratic in C_L and ϵ_0. If the coefficient of the $C_L\epsilon_0$ term is negative, there exists a C_L for minimum C_D for a given ϵ_0. If the term is positive, there exists an ϵ_0 for a maximum C_D for a given C_L. It is of interest to note that the value of ϵ_0 at this extremum is that which satisfies Eq. (12), the requirement for zero tail load. The negative $\epsilon_0{}^2$ term in Eq. (8) suggests the possibility of using large values of ϵ_0 (positive or negative) to reduce the trimmed drag.

Numerical Results

The foregoing equations, as well as some additional equations presented in the Appendix, were evaluated for a small twin-engine aircraft whose data were given in Ref. 8. Complete data for the wing-body configuration including the downwash at the tail were available. The flight configuration selected was the cruise configuration with flap angle and thrust coefficient equal to zero. The geometry and aerodynamic data are given in Table 1. For illustrative purposes two values of ϵ_0 were used, $\epsilon_0 = 0.0°$ and $\epsilon_0 = 3.67°$, the latter value determined from the wind-tunnel data and Eq. (5).

Figure 1 shows the tail load required for various c.g. positions including the optimal determined when $\epsilon_0 = 0°$ and $3.67°$. As indicated by Eq. (11), the tail load is constant for $\epsilon_0 = 0°$ and decreases and becomes negative for $\epsilon_0 = 3.67°$. The concave downward appearance of the curves for constant c.g. position occurs because of the negative C_{m0wb} [see Eq. (A3)]. Figure 2 shows the distribution of drag between the wing-body and tail. The negative contribution from the tail comes from the effect mentioned earlier ($C_{L_t}\epsilon$). Increased ϵ_0 increases this effect, rotating the drag curves in Fig. 2 counterclockwise about the zero tail drag point, causing a forward c.g. position for minimum C_D. Figure 3 shows the optimal c.g. position as given by Eq. (7). Also, the value of ϵ_0 and the corresponding c.g. position is shown for the ϵ_0 which satisfies Eq. (15).

Finally, the drag coefficients and the possible improvements due to c.g. placement are shown in Table 2.

Table 1 Aircraft parameters

	Constant parameters	
$A\!\!\!R_w = 6.98$	$h_0 = 0.25$	$S = 179.08$ ft^2
$A\!\!\!R_t = 5.33$	$h_t = 3.34$	$W = 5200$ lbs
$a_t = 0.06$/deg	$k_t = 0.119$	$C_{D_{0wb}} + C_{D_{0t}}E_t = 0.0318$
$\bar{C} = 5.13$ ft	$S_t = 54.25$ ft^2	
	$C_T = 0.0(0.28)$	$\delta_f = 0°$
$a_{wb} = 0.085$/deg (0.101)		$\epsilon_0 = 3.67°(2.98)$
$C_{m_{0wb}} = -0.05(-0.055)$		$\partial\epsilon/\partial\alpha = 0.33(0.567)$
$C_{m_{\alpha wb}} = 0.011$/deg (0.011)		$h_{nwb} = 0.12(0.141)$
$k_{wb} = 0.0518$		$\eta_t = 1(1.5)$

Table 2 Center of gravity effects on $C_D - C_{D0}$

Speed, fps	$h_0 = 0.05$	0.25	0.45	Opt($\epsilon = 0.0$)	% overall C_D reduction from $h_0 = 0.25$
150	0.0630	0.0601	0.0606	0.0599	0.22
200	0.0202	0.0191	0.0191	0.0189	0.39
250	0.0084	0.0079	0.0078	0.0078	0.25
300	0.0042	0.0038	0.0037	0.0037	0.28
350	0.0023	0.0021	0.0020	0.0020	0.29
	$C_{D0} = 0.0318$			Opt($\epsilon = 3.67$)	
150	0.0605	0.0619	0.0667	0.0604	1.60
200	0.0183	0.0196	0.0221	0.0181	2.92
250	0.0069	0.0079	0.0093	0.0062	4.28
300	0.0028	0.0036	0.0045	0.0018	5.08
350	0.0011	0.0016	0.0023	−0.0001	5.09

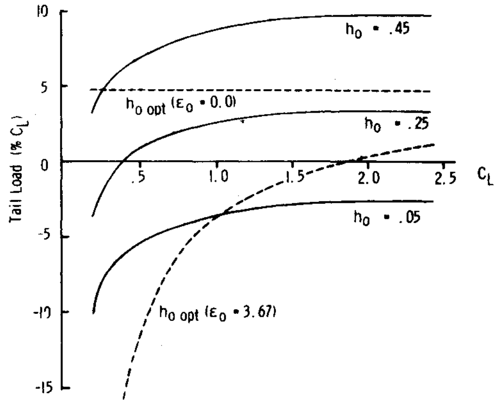

Fig. 1 Load distributions.

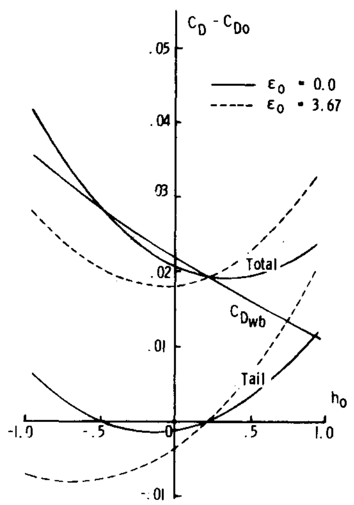

Fig. 2 Drag coefficient breakdown vs c.g. position.

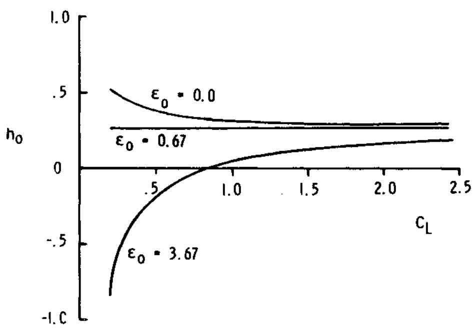

Fig. 3 Optimal c.g. positions for minimum C_D.

Generally, the improvements are small, as expected, but the potential for improvement increases with ϵ_0 as indicated by Eq. (8).

Closing Remarks

The foregoing analysis does not include propeller slipstream effects which could be considerably important for conventional small aircraft. The changes in the parameters for a C_T of 0.28 are indicated in Table 1 in parentheses. Corresponding curves, however, were not generated here.

The optimal load distribution is dependent only on the parameters which appear in Eq. (6). The optimal c.g. position depends on the same parameters plus $C_{m_{0wb}}$, h_t, and h_{nwb}. The analysis and subsequent numerical calculations indicate a strong sensitivity of optimal load distribution and corresponding c.g. movement to ϵ_0. In general, each flight speed requires a different optimal c.g. position, which, under certain circumstances, can be extremely far forward. It appears, however, that by proper design the optimal c.g. position can be made almost stationary and at a location which would yield a suitable stability margin. In particular, the proper ϵ_0 and $C_{m_{0wb}}$ pair would assure constant c.g. location, the position of which would be determined by h_t, h_{nwb}, k_{wb}, k_{wb}, and $\partial\epsilon/\partial\kappa$. Since these parameters are dependent on each other, the design problem to achieve specified values is a nontrivial exercise which is the subject of future investigation.

Appendix

The general equations which determine the load distribution between the tail and the wing-body are determined from Eqs. (1) and (2). These can be solved for $C_{L_{wb}}$ and $(C_L)_t$ to obtain

$$C_{L_{wb}} = \frac{(h_t - h_0) C_L - C_{m_{0wb}}}{h_t - h_{nwb}} \tag{A1}$$

$$(C_L)_t = \frac{(h_0 - h_{nwb}) C_L + C_{m_{0wb}}}{(h_t - h_{nwb})} \tag{A2}$$

or

$$\frac{(C_L)_t}{C_L} = \left(\frac{h_0 - h_{nwb}}{h_t - h_{nwb}} \right) + \left(\frac{C_{m_{0wb}}}{h_t - h_{nwb}} \right) \frac{1}{C_L} \tag{A3}$$

which is analogous to Eq. (11).

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